03/12/2018

This question paper contains 4 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **90** 12

Unique Paper Code : **32351301**

Name of the Paper : **Theory of Real Functions**

Name of the Course : **B.Sc. (Hons.) Mathematics**

Semester : **III**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *three* parts from each question

All questions are compulsory.

1. (*a*) Let $A \subseteq R$, $f : A \to R$ and $c \in R$ be a cluster point of A. Then prove that f can have only one limit at c. 5

(*b*) Use the $\in$-δ definition of the limit to prove that $\lim_{x \to c} x^3 = c^3$ for any $c \in R$. 5

(*c*) State divergence criterion for limit of a function. Show that $\lim_{x \to 0} (x + \operatorname{sgn}(x))$ does not exist. 5

P.T.O.

(*d*) Prove that :

(*i*) $\lim_{x \to 0+} \frac{1}{x} = \infty$

(*ii*) $\lim_{x \to \infty} \frac{1}{x^2} = 0.$ 5

2. (*a*) Let $A \subseteq R$, $f, g, h : A \to R$ and $c \in R$ be a cluster point of A. If $f(x) \le g(x) \le h(x)$ for all $x \in A$, $x \neq c$ and if $\lim_{x \to c} f(x) = L = \lim_{x \to c} h(x)$, then prove that $\lim_{x \to c} g(x) = L$. 5

(*b*) State and prove sequential criterion for continuity of a real valued function. 5

(*c*) Let the function $f : R \to R$ be defined by

$$f(x) = \begin{cases} 2x & : \text{if } x \text{ is rational} \\ x+3 & : \text{if } x \text{ is irrational} \end{cases}$$

Find all the points at which f is continuous. 5

(*d*) Let $x \to [x]$ denote the greatest integer function. Determine the points of continuity of the function $f(x) = x - [x]$, $x \in R$. 5

3. (*a*) Let f be a continuous real valued function defined on $[a, b]$. By assuming that f is a bounded function show that f attains its bounds on $[a, b]$. 5

(*b*) State Bolzano's Intermediate value theorem and show that the function $f(x) = xe^x - 2$ has a root c in the interval $[0, 1]$. 5

(*c*) Let $f : R \to R$ is continuous on R and suppose that $f(r) = 0$ for every rational numbers r. Show that $f(x) = 0$ for all $x \in R$. 5

(*d*) Define uniform continuity of a function. Prove that if a function is continuous on a closed and bounded interval I, then it is uniformly continuous on I. 5

4. (*a*) Show that the function $f(x) = 1/x^2$ is uniformly continuous on $A = [0, \infty[$ but it is not uniformly continuous on $B =]0, \infty[$. 5

(*b*) Determine where the following function $f : R \to R$ is differentiable, $f(x) = |x - 1| + |x + 1|$. 5

(c) Let f be defined on an interval I containing the point c. Then prove that f is differentiable at c if and only if there exists a function ϕ on I that is continuous at c and satisfies $f(x) - f(c) = \phi(c)(x - c)$ for all $x \in$ I. In this case, we have $\phi(c) = f'(c)$. Using the above result find the function ϕ for $f(x) = x^3$, $x \in$ R 5

(d) State and prove Mean Value Theorem. 5

5. (a) State Darboux's theorem. Suppose that $f : [0, 2] \to$ R is continuous on [0, 2] and differentiable on]0, 2[and that $f(0) = 0$, $f(1) = 1$, $f(2) = 1$. (*i*) Show that there exists $c_1 \in (0, 1)$ such that $f'(c_1) = 1$. (*ii*) Show that there exists $c_2 \in (1, 2)$ such that $f'(c_2) = 0$. (*iii*) Show that there exists $c \in (0, 2)$ such that $f'(c) = 1/10$. 5

(b) Let f : I $\to$ R be differentiable on the interval I. Then prove that f is increasing on I if and only if $f'(x) \geq 0$ for all $x \in$ I. 5

(c) State Taylor's theorem. Use it to prove that $1 - x^2/2 \leq \cos x$ for all $x \in$ R. 5

(d) Find the Taylor series for e^x and state why it converges to e^x for all $x \in$ R. 5

8/12/2018

This question paper contains 4 printed pages.

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Your Roll No. ....

S. No. of Paper : **91** **I**

Unique Paper Code : **32351302**

Name of the Paper : **Group Theory - I**

Name of the Course : **B.Sc. (Hons.) Mathematics**

Semester : **III**

Duration : **3 hours**

Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

*Attempt any **two** parts from each question.*
***All** questions are compulsory.*

1. (a) Define a group. Give an example of:

(i) an abelian group consisting of eight elements,

(ii) a non-abelian group consising of six elements,

(iii) an infinite abelian group, and

(iv) an infinite non-abelian group.

(b) Show that the set {5, 15, 25, 35} is a group under multiplication modulo 40. What is the identity element of this group? Find the inverse of each element.

(c) Prove that the intersection of an arbitrary family of subgroups of a group G is again a subgroup of G. What can you say about the union of two subgroups? Justify your answer. 2×6=12

P. T. O.

2. (a) (i) Prove that in (Z, +), the group of integers under addition, every non-zero element is of infinite order.

(ii) Let G be a group and $a \in G$. If $|a| = n$ and k is a positive divisor of n, then prove that $|a^{n/k}| = k$.

(b) Prove that the order of a cyclic group is equal to the order of its generator.

(c) Define a cyclic group. If $G = (a)$ is a finite cyclic group of order n, then prove that the order of any subgroup of G is a divisor of n, and for each positive divisor k of n, G has exactly one subgroup of order k, namely, $(a^{n/k})$.

2×6.5=13

3. (a) Prove that if the identity permutation $\varepsilon = \beta_1 \cdots \beta_r$ where the β's are 2-cycles then r is even.

(b) Show that for $n \geq 3$, $Z(S_n) = \{I\}$.

(c) Prove that:

(i) a group of prime order has no proper, non-trivial subgroup. State its converse. Is it true?

(ii) a group of prime order is cyclic and any non-identity element can be taken as its generator.

2×6=12

4. (a) Let G be a finite group of permutations of a set S. Then prove that for any i from S:

$$|G| = |orb_G(i)|\,|stab_G(i)|.$$

(b) (i) Prove that the center Z(G) of a group G is a subgroup of G and is normal in G.

(ii) If H is a subgroup of G such that H is contained in the center Z(G), then prove that H is a normal subgroup of G. Is the converse true? Justify your answer.

(c) Let N be a normal subgroup of a group G and let H be a subgroup of G. If N is a subgroup of H, prove that H/N is a normal subgroup of G/N if and only if H is a normal subgroup of G. 2×6.5=13

5. (a) Let **C** be the complex numbers and:

$$\mathbf{M} = \left\{ \begin{bmatrix} a & -b \\ b & a \end{bmatrix} : a, b \in R \right\}.$$

Prove that **C** and **M** are isomorphic under addition and $\mathbf{C}^* = \mathbf{C} \setminus \{0\}$ and $\mathbf{M}^* = \mathbf{M} \setminus \{0\}$ are isomorphic under multiplication.

(b) Prove that an infinite cyclic group is isomorphic to (Z, +). Hence show that every subgroup of an infinite cyclic group is isomorphic to the group itself.

(c) Let G be a group of permutations. For each σ in G, define

$$\operatorname{sgn}(\sigma) = \begin{cases} 1, & \text{if } \sigma \text{ is an even permutation,} \\ -1, & \text{if } \sigma \text{ is an odd permutation.} \end{cases}$$

Prove that sgn is a homomorphism from G to $\{1, -1\}$. What is the kernel? 2×6 =12

6. (a) Let ϕ be a homomorphism from a group G to a group $\bar{G}$. Let g be an element of G. Then:

   (i) $\phi(g^n) = \phi(g)^n$ for all $n \in \mathbf{Z}$.

   (ii) ϕ is one-one if and only if $\ker(\phi) = \{e\}$, where e is the identity of G.

(b) State and prove the First Isomorphism Theorem.

(c) (i) Suppose ϕ is a homomorphism from $U(30)$ to $U(30)$ and $\mathrm{Ker}(\phi) = \{1, 11\}$.

   If $\phi(7) = 7$, find all elements of $U(30)$ that map to 7.

   (ii) Let G be a group. Prove that the mapping $\phi(g) = g^{-1}$, for all $g \in G$, is an isomorphism from G onto G if and only if G is Abelian. 2×6.5=13

15/12/18

This question paper contains 4+2 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **92** 14

Unique Paper Code : **32351303** **I**

Name of the Paper : **C-7 Multivariate Calculus**

Name of the Course : **B.Sc. (Hons.) Mathematics**

Semester : **III**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All sections are compulsory.

All questions carry equal marks.

Section I

Attempt any *six* questions from this section.

1. Let f be the function defined by $f(x, y) = \dfrac{x^2 + 2y^2}{x^2 + y^2}$ for $(x, y) \neq (0, 0)$.

(*a*) Find $\lim\limits_{(x, y) \to (2, 1)} f(x, y)$.

(*b*) Prove that f has no limit at $(0, 0)$.

P.T.O.

2. The temperature at the point (x, y) on a given metal plate in the xy-plane is determined according to the formula $T(x, y) = x^3 + 2xy^2 + y$ degrees. Compute the rate at which the temperature changes with distance if we start at (2, 1) and move :

(*a*) parallel to the vector **j**.

(*b*) parallel to the vector **i**.

3. The Company sells two brands X and Y of a commercial soap, in thousand-pound units. If x units of brand X and y units of brand Y are sold, the unit price for brand X is $p(x) = 4{,}000 - 500x$ and for brand Y is $q(y) = 3{,}000 - 450y$.

(*a*) Find the total revenue R in terms of p and q.

(*b*) Suppose the brand X sells for \$ 500 per unit and brand Y sells for \$ 750 per unit. Estimate the change in total revenue if the unit prices are increased by \$ 20 for brand X and \$ 18 for brand Y.

4. If

$$w = f\left(\frac{r - s}{s}\right),$$

show that

$$r\frac{\partial w}{\partial r} + s\frac{\partial w}{\partial s} = 0.$$

5. Find the directional derivative of $f(x, y) = e^{x^2y^2}$ at P(1, –1) in the direction toward Q(2, 3).

6. Find the absolute extrema of $f(x, y) = 2\sin x + 5\cos y$ in the rectangular region with vertices (0, 0), (2, 0), (2, 5) and (0, 5).

7. Let $\mathbf{R} = x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}$ and $r = \|\mathbf{R}\|$, evaluate $\operatorname{div}\left(\frac{1}{r^3}\mathbf{R}\right)$.

Section II

Attempt any *five* questions from this section.

8. By using iterated integral, compute

$$\iint_R x\sqrt{1 - x^2}\, e^{3y}\, dA,$$

where R is the rectangle $0 \le x \le 1$, $0 \le y \le 2$.

9. Evaluate the double integral :

$$\iint_D \frac{dA}{y^2+1},$$

where D is the triangular region bounded by $y = -x$ and $y = 2$.

10. Evaluate the double integral

$$\int_0^2 \int_y^{\sqrt{8-y^2}} \frac{1}{\sqrt{1+x^2+y^2}}\, dx\, dy$$

by converting to polar co-ordinates.

11. Find the volume of the tetrahedron T bounded by the plane $2x + y + 3z = 6$ and the co-ordinates plane $x = 0$, $y = 0$ and $z = 0$.

12. Find the volume of the solid D bounded by the paraboloid $z = 1 - 4\left(x^2 + y^2\right)$ and the xy-plane.

13. Evaluate

$$\iint_D (x+y)^5 (x-y)^2\, dy\, dx$$

by using change of variable $u = x + y$ and $v = x - y$, where D is the region in the xy-plane which is bounded by the co-ordinate axes and the line $x + y = 1$.

Section III

Attempt any *four* questions from this section.

14. Evaluate the line integral

$$\int_C \mathbf{F} \,.\, d\mathbf{R},$$

where

$$\mathbf{F} = \frac{x}{\sqrt{x^2 + y^2}} i - \frac{y}{\sqrt{x^2 + y^2}} j$$

and C is the quarter circle path $x^2 + y^2 = a^2$, traversed from $(a, 0)$ to $(0, a)$.

15. Show that the vector field

$$\mathbf{F}(x, y, z) = \langle \sin z, -z \sin y, x \cos z + \cos y \rangle$$

is conservative and evaluate

$$\int_C \mathbf{F} \,.\, d\mathbf{R}$$

for any piecewise smooth path joining A(1, 0, −1) to B(0, −1, 1).

16. Use Green's theorem, to find the work done by the force field

$$\vec{F}(x, y) = (3y - 4x)\hat{i} + (4x - y)\hat{j}$$

when an object moves once counterclockwise around the ellipse $4x^2 + y^2 = 4$.

17. Use Stokes' theorem, to evaluate the line integral

$$\oint_C (3y\,dx + 2z\,dy - 5x\,dz)$$

where C is the intersection of the xy-plane and the hemisphere

$$z = \sqrt{1 - x^2 - y^2},$$

traversed counterclockwise as viewed from above.

18. Evaluate

$$\iint_S (\mathbf{F}.\mathbf{N})\,dS,$$

where $\mathbf{F} = x^2\hat{i} + xy\hat{j} + x^3y^3\hat{k}$ and S is the surface of the tetrahedron bounded by the plane $x + y + z = 1$ and the coordinate planes, with outward unit normal vector **N**.

Sl-No. of Q.P : 1689

2018

Unique Paper Code : 235301

Name of the Paper : Calculus-II (MAHT-301)

I

Name of the Course : B.Sc.(H)- Mathematics ~~II (Semester Scheme)~~

Semester : (II)

15

Duration : 3 hours

Maximum Marks : 75

Instruction for the candidates

(Write your Roll No. on the top immediately on the receipt of this question paper)

All sections are compulsory.

Attempt any five questions from each section. All questions carry equal marks.

(Section – 1)

1. Define level curves and sketch the level curves of the function $f(x, y) = 10 - x^2 - y^2$ cut by the planes $z = 1, z = 3$ and $z = 7$.
2. Show that the limit of the function defined by $f(x, y) = \frac{x^2y}{x^4+y^2}$ does not exist at $(x, y) = (0,0)$.
3. Compute the slope of the tangent line to the graph of the function $f(x, y) = x^2 \sin(x + y)$ at the point $P_0(\frac{\pi}{2}, \frac{\pi}{2}, 0)$ in the direction parallel to xz-plane.
4. Use an incremental approximation to estimate the function $f(x, y) = e^{xy}$ at the point $(x, y) = (1.01, 0.98)$.
5. Given the function $F(x, y) = x^2 + y^2$, where $x = u \sin v$ and $y = u - 2v$. Find the partial derivatives $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$ in two ways by considering $z = F(x(u, v), y(u, v))$.
6. Find all the relative extrema and saddle points of the function

$$f(x, y) = 2x^2 + 2xy + y^2 - 2x - 2y + 5.$$

(Section – 2)

7. Evaluate the iterated integral $\int_1^2 \int_0^{\pi} x \cos y \, dy \, dx$.

8. Find the volume of a tetrahedron T bounded by the plane $2x + y + 3z = 6$ and the coordinate planes.
9. Using cylindrical coordinates, find the volume of the solid in the first octant that is bounded by the cylinder $x^2 + y^2 = 2y$, the half-cone $z = \sqrt{x^2 + y^2}$, and xy – the plane.
10. Evaluate the triple integral $\int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{4}} \int_0^{\cos\varphi} \rho^2 \sin\varphi \; d\rho \, d\theta \, d\varphi$.
11. If $u = xy$ and $v = x^2 - y^2$ express the Jacobian $\frac{\partial(x,y)}{\partial(u,v)}$ in terms of u and v .
12. Evaluate $\int_0^2 \int_y^{\sqrt{(8-y^2)}} \frac{dx\,dy}{\sqrt{(1+x^2+y^2)}}$ using polar coordinates.

(Section – 3)

13. Evaluate the line integral $\oint_C x^2 z \, ds$, where C is the helix $x = \cos t, y = 2t, z = \sin t$, for $0 \le t \le \pi$.
14. a) Find the divergence of the function $\boldsymbol{F} = xy\,\boldsymbol{i} + yz\,\boldsymbol{j} + xz\,\boldsymbol{k}$.

curl

b) Let $\boldsymbol{R} = (x, y, z)$ and $r = \|\boldsymbol{R}\| = \sqrt{x^2 + y^2 + z^2}$. Show that $url\left(\frac{1}{r^3}\,\boldsymbol{R}\right) = 0$.
15. Find the work done by an object moving along the C in the force field $\boldsymbol{F}(x, y) = (x + xy^2)\hat{\imath} + 2(x^2 y - y^2 \sin y)\hat{\jmath}$, where C is the closed path in the plane defined by the curve $y = x^2$ from $(0,0)$ to $(1,1)$, followed by the lines $y = 1$ and $x = 0$ from $(1,1)$ to $(0,1)$ and from $(0,1)$ to $(0,0)$ respectively.
16. Compute the flux integral $\iint_S \boldsymbol{F}.\boldsymbol{N}\, dS$ where $\boldsymbol{F} = xy\,\boldsymbol{i} + z\,\boldsymbol{j} + (x + y)\boldsymbol{k}$ and S is the triangular surface cut off from the plane $x + y + z = 1$ by the coordinate planes. Assume $\boldsymbol{N}$ is the upward unit normal.
17. Use divergence theorem to evaluate the surface integral $\iint_S \boldsymbol{F}.\boldsymbol{N}\, dS$ where $\boldsymbol{F} = x\,\mathrm{i} + y\,\boldsymbol{j} + z\,\boldsymbol{k}$ and S is the closed bounded surface $0 \le x \le 1, 0 \le y \le 1, 0 \le z \le 1$, and assume $\boldsymbol{N}$ is the outward unit normal vector.
18. Let S be the portion of the plane $x + y + z = 1$ that lies in the first octant, and let C be the boundary of S , traversed counterclockwise as viewed from the above. Use Stoke's theorem to evaluate $\oint_C \boldsymbol{F}.d\boldsymbol{R}$ where $\boldsymbol{F} = -\frac{3}{2}y^2\,\boldsymbol{i} - 2xy\,\boldsymbol{j} + yz\,\boldsymbol{k}$.

SET B

2018

This question paper contains 2 printed pages

Roll No.

Sl-No 07 Q.P: 1691

Unique paper code : 235304 (16)

I

Name of the course : B. Sc. (Hons) Mathematics

Name of the paper : MAHT 303 – Algebra II

Semester : III

Duration : 3 Hours

Maximum marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of the question paper.
2. Attempt any ***two*** parts from each question.
3. All questions are compulsory.

1\. (a) Let H be a finite nonempty subset of a group G. Then, prove that H is a subgroup of G if and only if H is closed under the binary operation of G.

(b) Suppose that $|a| = 24$. Find a generator for $< a^{21} > \cap < a^{10} >$. In general, find a generator for the subgroup $< a^m > \cap < a^n >$.

(c) Let G be a finite cyclic group of order n. Prove that for every divisor d of n, there exists a unique subgroup of order n.

(6, 4+2, 6)

2\. (a) Prove that every group of order 3 is Abelian.

(b) In a group G, prove that, for all $a, b \in G$, the equations $ax = b$ and $ya = b$ have unique solutions in G.

(c) In a G group, let $a, b \in G$ such that $(ab)^2 = a^2b^2$. Then, prove that $ab = ba$.

(6,6,6)

3\. (a) Prove that the disjoint cycles commute.

(b) Define a cyclic group. Prove that every group of order p, where p is a prime, is cyclic.

(c) Prove that the set A_n of even permutations of the group S_n is a normal subgroup of S_n and $|A_n| = |S_n|/2$, where $n \geq 3$.

(6,6,6)

4. (a) State and prove Lagrange's Theorem.

(b) Define a coset of a subgroup and a normal subgroup of a group G. Prove that a sub-group H of G is normal if and only if $xHx^{-1} \subseteq H$.

(c)(i) Let H be a subgroup of a group G having the index 2. Then, prove that H is a normal subgroup of G.

(ii) Let H be a subgroup of a group G such that $x^2 \in H$, for all $x \in G$. Then, prove that H is a normal subgroup of G.

$(2+4\frac{1}{2}, 2+4\frac{1}{2}, 4+2\frac{1}{2})$

5. (a) Let G be a group. Then, prove that $\dfrac{G}{Z(G)} \cong Inn(G)$.

(b) (i) Find $Aut(\mathbb{Z})$.

(ii) Let G be a group and φ be a mapping from G to G defined by $\varphi(g) = g^{-1}$. Then, prove that φ is an automorphism if and only if G is Abelian.

(c) Let φ be a homorphism from a group G to a group $\bar{G}$ and let $g \in G$. If $\varphi(g) = \bar{g}$, then prove that $\varphi^{-1}(\bar{g}) = g \ker \varphi$.

$(6\frac{1}{2}, 3+3\frac{1}{2}, 6\frac{1}{2})$

6. (a) Prove that every group is isomorphic to a group of permutations.

(b) Prove that any finite cyclic group of order n is isomorphic to Z_n and any infinite cyclic group is isomorphic to $\mathbb{Z}$.

(c) Let φ be a homorphism from a group G onto a group $\bar{G}$. Then, prove that

$$\frac{G}{\ker \varphi} \cong \bar{G}.$$

$(6\frac{1}{2}, 6\frac{1}{2}, 6\frac{1}{2})$

Roll No. ...2018........

S. No. of Question Paper: 1852

17

Unique Paper Code : **2351303**

Name of the Paper : **Numerical Methods**

Name of the Course : **B.Sc. (H) Mathematics Un~~der Erstwhile FYUP~~**

Semester : **III**

Duration: **3 Hours** Maximum Marks: 75

(Write your Roll No. on the top immediately on receipt of this question paper)

All questions are compulsory

Attempt any two parts from each question

Marks are indicated against each question

Use of Scientific Calculator is allowed

1. (a) Perform four iterations of the Newton Raphson method to obtain a root of $f(x) = x^3 - 5x + 1$ taking $p_0 = 0.5$.

(b) Define the order of convergence of an iterative method $\{x_n\}$. Determine the order of convergence of the recursive scheme $x_{n+1} = \frac{1}{2}(x_n + b/x_n)$.

(c) Describe the Bisection method to approximate the root of an equation. Perform three iterations of this method to approximate a zero of $f(x) = e^{-x} - x$ on the interval $[0,1]$. 13

2. (a) Derive the rate of convergence of the Regula Falsi method.

(b) Perform three iterations of the Regula Falsi method to determine a positive root of $f(x) = x^5 + 2x - 1$ in the interval $[0,1]$.

(c) Find the approximate root of $f(x) = x^3 + 2x^2 - 3x - 1$ by secant method, taking

$p_0 = 2$ and $p_1 = 1$, until the error $|p_n - p_{n-1}| < 5 \times 10^{-3}$. 12

3. (a) Find an LU decomposition of the matrix:

$$A = \begin{bmatrix} 4 & 1 & 1 \\ 1 & 4 & -2 \\ 3 & 2 & -4 \end{bmatrix}$$

and use it to solve the system $AX = [4 \quad 4 \quad 6]^T$.

(b) Perform three iteration of Gauss-Seidel method to solve the system of linear equations $AX = b$, where

$$A = \begin{bmatrix} -3 & 1 & 0 \\ 2 & -3 & 1 \\ 0 & 2 & -3 \end{bmatrix}, b = \begin{bmatrix} -2 \\ 0 \\ -1 \end{bmatrix}$$

Take $\boldsymbol{X}^{(0)} = \boldsymbol{0}$ as the initial approximation.

(c) Perform three iterations of Jacobi method to solve the system of linear equations

$$10x + 4y - 2z = 12$$

$$x - 10y - z = -10$$

$$5x + 2y - 10z = -3$$

Take $\boldsymbol{X}^{(0)} = [\boldsymbol{1} \quad \boldsymbol{1} \quad \boldsymbol{1}]$ as the initial approximation. 13

4. (a) For the following data

X	-1	2	4	5
f(x)	-5	13	255	625

obtain the Lagrange interpolating polynomial. Estimate f(3).

(b) Calculate the nth divided difference of 1/x, based on the points $x_0, x_1, \ldots, x_n$.

(c) For the function f(x)=sin x, obtain the Lagrange linear interpolating polynomial in the interval [1,3]. Obtain approximate values of f(1.5) and f(2.5). 12

5. (a) Define the forward difference operator Δ, central difference operator δ and average operator μ. Prove that:

(i) $\delta = \Delta(1+\Delta)^{-1/2}$

(ii) $\mu = \left(1+\frac{\Delta}{2}\right)(1+\Delta)^{1/2}$

(b) For the following data, obtain the forward and the backward Newton difference polynomials and interpolate at x=0.25 and x=0.35.

x	1	1.5	2.0	2.5
f(x)	2.7183	4.4817	7.3891	12.1825

(d) If $x_0, x_1, \ldots, x_n$ are n+1 distinct points and f is defined at these n+1 points, then prove that interpolating polynomial P, of degree at most n, is unique.

13

6. (a) Find approximate value of $\int_0^1 \frac{1}{1+x^3}dx$ using composite Trapezoidal rule with 2 equal sub-intervals.

(b) Find approximate value of $\int_0^1 tan^{-1}x\,dx$ using Simpson's rule. Verify that the theoretical error bound is satisfied.

(c) Apply Euler's method to approximate the solution of the initial value problem

$$\frac{dx}{dt} = t^2 - 2x^2 - 1, \quad 0 \le t \le 1, \quad x(0) = 1$$

over the interval [0,1] using four steps.

12